Quantum engineering of photon states with entangled atomic ensembles

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We propose and analyze a new method to produce single and entangled photons which does not require cavities. It relies on the collective enhancement of light emission as a consequence of the presence of entanglement in atomic ensembles. Light emission is triggered by a laser pulse, and therefore our scheme is deterministic. Furthermore, it allows one to produce a variety of photonic entangled states by first preparing certain atomic states using simple sequences of quantum gates. We analyze the feasibility of our scheme, and particularize it to: ions in linear traps, atoms in optical lattices, and in cells at room temperature.

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The deterministic generation of collimated single and entangled photons is of crucial importance in Quantum Information, like in quantum cryptography [1], quantum computation [2], quantum lithography [3] or quantum interferometry [4, 5]. Most of the methods tested so far require high-Q cavities, something which is very demanding in practice [6, 7, 8, 9]. The engineering of quantum states in atomic systems is now possible thanks to the experimental progress experienced by the field of Atomic Physics during the last years. In fact, with trapped ions it has been already possible to create so–called W [10] and GHZ [11] states of up to 8 ions. At the same time, scientists have been able to produce other kinds of entangled states [12] with atoms in optical lattices. Furthermore, with the advent of Rydberg techniques [13] it will soon be possible to create W–like states in that system or in atomic ensembles at room temperature. Apart from their fundamental interest, some of those states may have applications in precision spectroscopy [14, 15].

In this work we show that the ability of creating those atomic states may have a strong impact in different subfields of quantum information, as it may lead to a very efficient way of creating certain kind of entangled photonic states which are required in various applications. The main idea is to use a laser and an internal level configuration such that we can map the atomic state onto photonic states corresponding to modes propagating in a well defined direction. Our scheme uses the well known fact [16, 17] that, under certain circumstances, light scattering takes place predominantly in the forward direction due to an interference effect. In fact, this effect is the basis of one of the building blocks of the repeater scheme proposed in [18], and has been recently demonstrated in a series of experiments [19, 20, 21]. There, a single excitation is created in an atomic ensemble by detecting a photon emission in a certain direction. Then, the excitation is released in the forward direction by using a laser. Building on this fact, we propose to create certain kind of excitations by using quantum gates or atomic interactions, which give rise to the desired entangled states when they are released using a laser, and which propagate in the desired direction due to the mentioned interference effect.

Let us consider a set of N atoms with (ground) hyperfine levels $|g\rangle$ and $|s_{a,b}\rangle$ (see Fig. 1 (a)). We consider states of the form

$$|\mathbf{k}_a^{(n_a)}, \mathbf{k}_b^{(n_b)}\rangle = \frac{1}{\sqrt{n_a!\, n_b!}} \left(\sigma^\dagger_{a,\mathbf{k}_a}\right)^{n_a} \left(\sigma^\dagger_{b,\mathbf{k}_b}\right)^{n_b} |0\rangle, \quad (1)$$

and linear combinations thereof. Here, $|0\rangle = |g\rangle_1 \ldots |g\rangle_N$, and

$$\sigma^\dagger_{x,\mathbf{k}_\alpha} = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} e^{-i\mathbf{k}_x \mathbf{r}_j^0} \sigma^\dagger_{x,j}, \quad x = a, b, \quad (2)$$

where $\sigma^\dagger_{x,j}$ excites an atom from $|g\rangle_j$ to $|s_x\rangle_j$, and $\mathbf{r}_j^0$ are the equilibrium position of the atoms. In the limit $n_x \ll N$, Eq. (1) defines a set of orthonormal collective states with n_x atoms excited in $|s_x\rangle$ and linear momentum $\mathbf{k}_x$. Those states can be indeed readily created using trapped ions or Rydberg techniques (see Appendix A1,3).

In order to release the photons, one sends a laser pulse of wavevector $\mathbf{k}_\mathrm{L}$ which couples level $|s_x\rangle$ to some electronically excited ones $|e_x\rangle$, respectively. The large population of level $|g\rangle$ together with the initial entanglement (coherences) between the atoms, will now stimulate the emission of photons from the excited states to the level $|g\rangle$, which overall will produce the mapping between these states and the photonic states,

$$|\mathbf{k}_a^{(n_a)}, \mathbf{k}_b^{(n_b)}\rangle \to |n_a\rangle_{\mathbf{k}_a+\mathbf{k}_\mathrm{L},\sigma_a} |n_b\rangle_{\mathbf{k}_b+\mathbf{k}_\mathrm{L},\sigma_b}; \quad (3)$$

that is, (1) is mapped to a Fock state of n_x photons with momenta $\mathbf{k}_x + \mathbf{k}_\mathrm{L}$ and polarization σ_x, where σ_x is the polarization of the light in each decay channel. Moreover, due to the linearity of this process, superpositions of states of the form (1) will be mapped onto superpositions of photonic states (3). For example, the atomic state $\left(|\mathbf{k}^{(1)}, \mathbf{q}^{(1)}\rangle + |\mathbf{q}^{(1)}, \mathbf{k}^{(1)}\rangle\right)/\sqrt{2}$ will emit a pair of entangled photons in different directions. The mapping (3) is strictly valid under ideal conditions, and in the limit $N \to \infty$, and the directionality in the photon emission is directly connected to the momentum conservation which, in turn, is a consequence of the constructive interference in the field emitted by each atom. Thus, the crucial issue in our scheme is to determine how this mapping is

modified in finite atomic ensembles under nonideal conditions. In the following we analyze such questions in detail, concentrating in the simplest case in which we have a single excitation with momentum $|\mathbf{k}_0\rangle$ in $|s_a\rangle$ (i.e. our initial state is a W-like state) and thus we produce a single photon. We determine a function $f(\Omega)$, which is proportional to the probability density that the photon is emitted in the direction Ω. In general, $f = f_{\rm coh} + f_{\rm inc}$; that is, it is the sum of a coherent contribution and an incoherent one. The later appears whenever the positions of the particles fluctuate. $f_{\rm coh}$ contains the forward scattering contribution, which is emitted in a cone with a width $\Delta\Omega$ that decreases with the number of particles. $f_{\rm inc}$, on the contrary, describes isotropic light emission, thus, even when the light emitted in $\Delta\Omega$ is collected, the contribution $f_{\rm inc}$ leads to a limitation in the efficiency of the setup. To quantify the error probability, we define

$$\mathcal{E} = \frac{\int d\Omega\ f_{\rm inc}(\Omega)}{\int d\Omega\ f(\Omega)}. \tag{4}$$

As long as the number of excited atoms is small $n_x \ll N$, this analysis can be easily generalized to the emission of states with many photons (1). One obtaines that the overall error is bounded by $1-(1-\mathcal{E})^{n_a+n_b}$.

The emission pattern can be obtained by studying the Heisenberg equations of motion of the field operators. The calculation inolves the study of the decay of the atomic state under collective effects (see Appendix C). To simplify our analysis we ignore the dipole pattern, in which case we get:

$$f(\Omega) = \frac{1}{N}\sum_{i,j=1}^{N} \langle e^{-i(k_{\rm L}\mathbf{n}_\Omega - \mathbf{k}_{\rm L})(\mathbf{r}_i - \mathbf{r}_j)}\rangle e^{i\mathbf{k}_0(\mathbf{r}_i^0 - \mathbf{r}_j^0)}. \tag{5}$$

$\mathbf{r}_j$ are the coordinate operators of the atoms, and thus Eq. (5) allows us to describe fluctuations in the position of the particles during the emission of light. In the following, we will show three different experimental set–ups where our scheme can be implemented. In order to analyze the performance in each of them, we first particularize the above formula to three different situations which are directly connected with those set–ups. We will focus on the angular width of the forward–scattering cone, $\Delta\Omega$, which measures the collimation of the emitted photons, and the error probability, $\mathcal{E}$, as figures of merit. Then we will introduce the possible implementations and will use those formulas to specify the conditions for them to correctly operate.

(i) Fixed atomic positions. In the case of a square lattice of particles trapped in 3D (see Fig. 1 (b)), the emission pattern is given by

$$f_0(\Omega) = \frac{1}{N}\prod_{\alpha=x,y,z}\frac{\sin^2((k_{\rm L}\mathbf{n}_\Omega^\alpha - (\mathbf{k}_{\rm L}+\mathbf{k}_0)^\alpha)d_0 N_\alpha/2)}{\sin^2((k_{\rm L}\mathbf{n}_\Omega^\alpha - (\mathbf{k}_{\rm L}+\mathbf{k}_0)^\alpha)d_0/2)}, \tag{6}$$

with N_α the number of atoms in each direction. $f_0(\Omega)$ has a series of diffraction peaks, which are reduced to a single one if $d_0 < \lambda/2$. In this regime, the emission is centered in a cone with $\mathbf{n}_\Omega$ in the direction of $\mathbf{k}_{\rm L}+\mathbf{k}_0$. Note that for simultaneous energy and momentum conservation condition $|\mathbf{k}_{\rm L}+\mathbf{k}_0| = k_{\rm L}$ has to be fulfilled. Since the positions of the atoms do not fluctuate, f_0 has only a coherent contribution ($\mathcal{E}=0$), and the only limitation for the effiency of the setup is the width of the emission cone, which scales in 3D like $\Delta\theta_{\rm 3D} \approx 1/(N^{1/3}k_{\rm L}d_0)$. In the case of a chain of atoms (1D) momentum is conserved only along the direction of the chain. Photon emission can be still directed efficiently along the axis of the chain, in a cone whose width scales like $\Delta\theta_{\rm 1D} \approx 1/\sqrt{Nk_{\rm L}d_0}$.

(ii) Fluctuating atomic positions. Let us consider a lattice of atoms at temperature T, trapped by independent harmonic potentials. The emission pattern is now the sum the of two contributions,

$$f_{\rm coh}(\Omega) = f_0(\Omega)g_T(\Omega),\ \ f_{\rm inc}(\Omega) = 1 - g_T(\Omega). \tag{7}$$

$g_T(\Omega) = e^{-((k_{\rm L}\mathbf{n}_\Omega - \mathbf{k}_{\rm L})\xi_T)^2}$, and ξ_T is the vector whose components are the size of the position fluctuations in each spatial direction, $(\xi_T^\alpha)^2 = x_0^2(1+2n_T^\alpha)$, with x_0^α the size of the ground state in harmonic potential, and n_T^α the number of motion quanta at T. Light scattered into $f_{\rm inc}$ represents an important fraction whenever $\xi_T^\alpha \gg d_0$. In this case, the emission of light is centered around $\mathbf{k}_{\rm L}$, since the uncertainty in the position of the particles averages out the intial linear momentum $\mathbf{k}_0$. The scaling of $\mathcal{E}$ in this regime strongly depends on the dimensionality of the system. In particular, in the case of a chain of atoms, $\mathcal{E}_{\rm 1D} = d_0/\lambda$, whereas in the square 3D lattice, we get $\mathcal{E}_{\rm 3D} \approx 12.6(d_0/\lambda)^2 N^{-1/3}$.

(iii) Statistical distribution of particles. Consider an ensemble of atoms (see Fig. 1 (c)), which move inside a square box of size L, such that their motion is faster than their radiative decay, that is, their average velocity v is such that $vL \gg \Gamma$, with Γ the emission rate. This situation can be described by assuming that the atoms are in a statistical distribution with equal probability to be at any point in the box. The situation is thus similar to that of a thermal state,

$$f_{\rm coh}\,(\Omega) = N g_{\rm box}(\Omega),\ \ f_{\rm inc}\,(\Omega) = 1 - g_{\rm box}(\Omega), \tag{8}$$

where

$$g_{\rm box}(\Omega) = \prod_{\alpha=x,y,z} \mathrm{sinc}^2\left(L(k_L\mathbf{n}_\Omega^\alpha - \mathbf{k}_{\rm L}^\alpha)\right). \tag{9}$$

Defining the average distance between particles like $d_0 = LN^{-1/3}$, we find the same scalings of $\Delta\theta$ as in case (i), and of $\mathcal{E}$, as in case (ii). Trapping schemes for atomic ensembles are simpler to realize but face the difficulty that conditions for the directionality of photon emission are more stringent. In the case of a lattice of particles at fixed positions, forward–scattering is ensured whenever condition $d_0 < \lambda/2$ is fulfilled. On the contrary, in the case of atomic ensembles, the incoherent contribution $f_{\rm inc}$ has to be small enough such that $\mathcal{E} \ll 1$, which implies

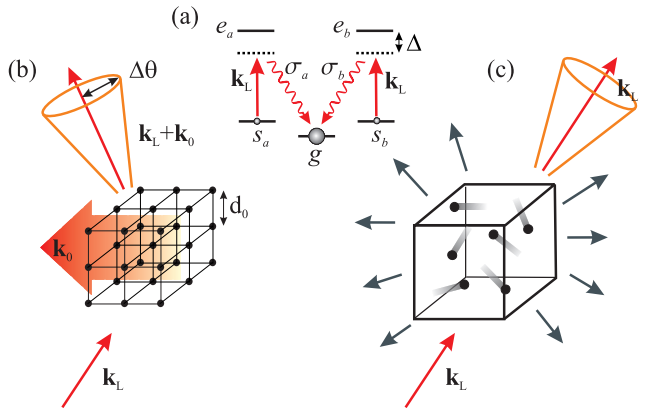

FIG. 1: (a) Level configuration for the release of atomic entangled states in photonic channels. (b) Release of a collective state with linear momentum $\mathbf{k}_0$, that has been generated in a lattice of atoms. (c) Emission of photons from an atomic ensemble, which consists of an incoherent contribution (isotropic), and a coherent one in the forward–scattering direction.

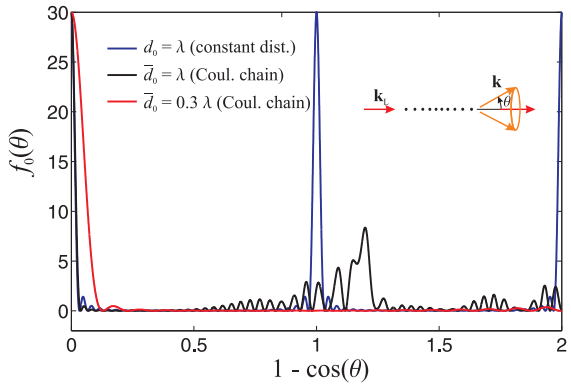

FIG. 2: Probability of photon emission from an ion chain with $N = 30$ ions initially in a W–state. The blue line corresponds to a chain with equally spaced ions with two diffraction peaks. Black and red lines corresponds to an ion Coulomb chain, in which ions are in an overall trapping potential and thus are not equally spaced. However, in the case that the average distance, $\bar{d}_0$ is small enough, light is also preferentially emitted in the forward–scattering direction.

$d_0 \ll \lambda$ in 1D, or, alternatively, a number of particles large enough in 3D.

Now we introduce three experimental set–ups where our scheme can be implemented. In the Appendix we show how to create the atomic states that we are considering here.

Trapped ions. This system is ideally suited to create collective states like (1), as was demonstrated recently in ref. [10]. Most usually ions are arranged in chains, such that we deal with the 1D situation discussed above. Even though trapped ions are not equally spaced, under the condition $\bar{d}_0 < \lambda/2$, with $\bar{d}_0$ the average distance, we still get light emission in the forward–scattering cone only, see Fig. 2. Considering two different internal levels, which can correspond to different states in an hyperfine multiplet, states such as those defined by Eq. (1) can be created by a number of quantum operations that scales linearly with the number of ions N (see A1). For example, the state $1/\sqrt{2}\,(|\mathbf{0}, 2\mathbf{k}_L\rangle + |2\mathbf{k}_L, \mathbf{0}\rangle)$, would emit two photons in the forward and backward directions along the chain axis, entangled in polarization. The main difficulty for the implementation of this idea with ions lies on the fact that ion–ion distances are usually in the range of a few μm, and thus condition $d_0 < \lambda/2$ is not fulfilled when considering optical wavelengths. A way out of this problem is to use optical transitions which lie in the range of $\lambda \gtrsim 5\mu m$, which can be found in ions such as Hg^+, Ba^+, or Yb^+ [22].

Cold atoms in optical lattices. By using optical lattices we fulfill the need of placing atoms at interparticle distances comparable to optical wavelengths, since potential wells in a standing–wave are indeed separated by $d_0 = \lambda_{sw}/2$, with λ_{sw}, the wavelength of the counterpropagating lasers. By using an optical transition such that $\lambda > \lambda_{sw}$, we are in the regime in which light emission is focused into a single Bragg peak. Although one could think of peforming quantum gates between ultracold neutral atoms to generate collective atomic states [23], this procedure faces the difficulties of quantum computation in this system, like for example how to achieve single atom addressability. More efficiently, one could avoid the use of quantum gates by using the dipole–blockade mechanism with Rydberg atoms, which allows us to generate W-states, as well as states which emit Fock states with a number M of photons [13] (see Appendix 3).

Atomic ensembles at room temperature. The very same techniques which can be applied to Rydberg atoms in an optical lattice can also be used in the case of hot ensembles. On the one hand, this setup has the advantage that atoms do not need to be cooled and placed in an optical lattice. On the other hand, it can be described by a statistical distribution of particles, and thus suffers from the fact that high efficiency in the release of photons is achieved under more severe conditions of particle density and atom number, as discussed above. However, densities which are high enough to fulfill the requirement $\mathcal{E} \ll 1$ have been recently reported in [24].

In conclusion, we have proposed to use current techniques for quantum engineering to generate atomic multipartite entangled states which can be efficiently mapped into photonic states. Our proposal relies on the release of spin–wave like excitations into a given spatial direction by means of interference effect, and can be implemented with trapped ions, atoms in optical lattices, and atomic ensembles at room temperature.

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APPENDIX A: CREATION OF COLLECTIVE ATOMIC STATES IN A CHAIN OF ATOMS

Entangled states of the form (1) and their linear combinations can be generated in a chain of particles, for example, of trapped ions, by means of a limited number of quantum operations. To demonstrate this, we first show that they can be written as Matrix Product States with a small bond dimension D, i.e. they can be written as

$$|\Psi\rangle = \sum_{i_1,\ldots,i_N} \langle \Phi_{\rm F}|V_{[N]}^{i_N}\ldots V_{[1]}^{i_1}|\Phi_{\rm I}\rangle |i_1\rangle\ldots|i_N\rangle, \qquad \text{(A1)}$$

In (A1), the indices $i_j = g, s_a, s_b$, and $V_{[j]}^{i_j}$ are $D\times D$ matrices acting on an auxiliary D–dimensional Hilbert space. D is given by the number of states which appear in the singular value decomposition (s.v.d.) of $|\Psi\rangle$ at any site in the chain [25]. As it is shown in [26], the state (A1) can be prepared by performing N gates which act on $[\log_2 D]+1$ qubits. Thus, as long as D is independent of N, the number of gates to be applied scales linearly with the total number of atoms.

To evaluate D, consider first the case of a state like (1) with atoms excited in level s_a only, and a partition of the chain in two parts L and R. We get n_a+1 states in the s.v.d. with respect to this partition, which correspond to states with a number of excited atoms in part L, ranging from 0 to n_a. This result is easily generalized to a linear combination of M states of the form (1), in which case we get $D = M(n_a+1)(n_b+1)$. For example, an entangled state of the form:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\mathbf{k}^{n_a=1},\mathbf{q}^{n_b=1}\rangle + |\mathbf{q}^{n_a=1},\mathbf{k}^{n_b=1}\rangle\right), \qquad \text{(A2)}$$

has $D=8$.

APPENDIX B: QUANTUM STATE ENGINEERING WITH RYDBERG BLOCKADE

Interactions between excited atomic states, like those that take place in Rydberg atoms, can be used to the create the states defined by Eq. (A2). This can be achieved in a single experimental step, without the need for quantum gates, if the proper configuration of atomic interactions is chosen. As an example, consider the 3 level configuration shown in Fig. 1 (a), and interactions between excited states such that atoms in levels $|s_a\rangle$, $|s_b\rangle$, interact strongly only if they are in the same excited state, that is, $U_{aa} = U_{bb} = U$, but $U_{ab} = 0$. We apply two lasers with wavectors $\mathbf{k}_{1,2}$ and Rabi frequencies $\Omega_{1,2}$, detuned with respect to the $|g\rangle - |s_{a,b}\rangle$ transition, such that $\Delta_1 = -\Delta_2 = \Delta$. If condition $\Delta_{1,2} \gg \Omega_{1,2}$ is fulfilled, then the lasers induce a two–photon transition with Rabi frequency $\Omega_{\rm eff} = \Omega_1\Omega_2/\Delta$. Furthermore, if $\Omega_{\rm eff} \ll U$, states with two atoms in the same excited state are not populated. Under these conditions there are two possible excitation channels, depicted in Fig. 3, which give rise to the linear combination (A2).

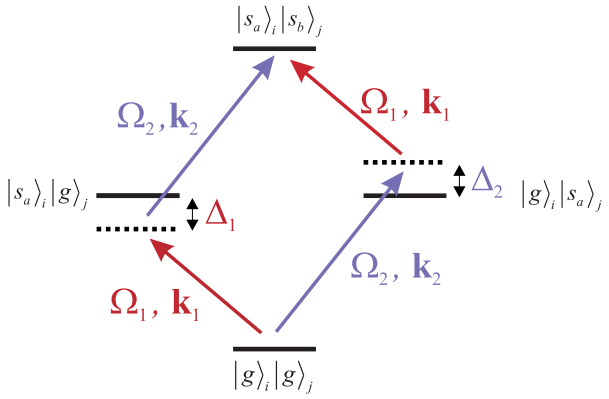

FIG. 3: Lasers and level configuration for the creation of atomic entangled states which emit pairs of photons entangled in polarization.

APPENDIX C: CALCULATION OF THE PHOTON DISTRIBUTION

We consider for simplicity the lambda configuration depicted in Fig. 1 (a), considering a single excited state $|s\rangle$, and a single auxiliary level $|e\rangle$. The interaction of the quantized electromagnetic field with the ensemble of atoms, after the adiabatic elimination of level $|e\rangle$, is described by

$$\begin{aligned} H_{\rm I} &= \sum_{j,\mathbf{k},\lambda} g_{\mathbf{k}\lambda}\left(\sigma_j^\dagger a_{\mathbf{k},\lambda} e^{i(\mathbf{k}-\mathbf{k}_L)\mathbf{r}_j + i\omega_L t} + h.c.\right), \\ g_{\mathbf{k},\lambda} &= \frac{\Omega_L}{2\Delta}\sqrt{\frac{\hbar\omega_k}{2\epsilon_0 V}}\left(\epsilon_{\mathbf{k}\lambda}\cdot\mathbf{d}_{ge}\right), \end{aligned} \qquad \text{(C1)}$$

σ_j refers to the $|g\rangle - |s\rangle$ atomic transition, Ω_L and $\mathbf{k}_{\rm L}$ are the Rabi frequency and wave–vector of the classical field, respectively, ω_k is the photon energy, $\epsilon_{\mathbf{k},\lambda}$ are the polarization vectors, and $\mathbf{d}_{ge}$ is the dipole matrix element for the $|g\rangle - |e\rangle$ transition.

The probability of photon emission is proportional to the the diagonal elements of the one–photon density matrix, which are obtained from the Heisenberg equation of motion for the field operators,

$$\begin{aligned} \langle a_{\mathbf{k}}^\dagger a_{\mathbf{k}}\rangle = \frac{1}{N}\sum_\lambda g_{\mathbf{k},\lambda}^2 \int_0^\infty d\tau_1 d\tau_2 e^{-i(\omega_k-\omega_L)(\tau_1-\tau_2)} \\ \sum_{ij}\langle e^{-i(\mathbf{k}-\mathbf{k}_L)(\mathbf{r}_i-\mathbf{r}_j)}\rangle\langle\sigma_i^\dagger(\tau_1)\sigma_j(\tau_2)\rangle. \end{aligned} \qquad \text{(C2)}$$

Since we are interested in the conditions for momentum conservation due to interference effects, we consider the

following atomic initial state,

$$|\mathbf{k}_0\rangle = \sigma^\dagger_{\mathbf{k}_0}|0\rangle, \;\; \sigma^\dagger_{\mathbf{k}_0} = \frac{1}{\sqrt{N}}\sum_{j=1}^{N} e^{-i\mathbf{k}_0\mathbf{r}^0_j}\sigma^\dagger_j. \qquad \text{(C3)}$$

The emission pattern depends thus on the two–time atomic correlation function, which in turns can be evaluated by means of a master equation which describes the decay of the atomic levels. In the case of the initial atomic state $\mathbf{k}_0$ (C3), fixed atom positions, and neglecting boundary effects, this correlation function can be evaluated exactly,

$$\langle\sigma^\dagger_i(\tau_1)\sigma_j(\tau_2)\rangle = e^{-\Gamma_{\mathbf{k}}(\tau_1+\tau_2)/2}e^{i\mathbf{k}_0(\mathbf{r}^0_i-\mathbf{r}^0_j)}, \qquad \text{(C4)}$$

where we have neglected an energy shift due to dipole–dipole interactions. Integrating (C2) over the absolute value of $\mathbf{k}$ yields the probability of photon emission,

$$I(\Omega) = \bar{I}(\Omega)f(\Omega). \qquad \text{(C5)}$$

$\bar{I}(\Omega)$ is the dipole pattern,

$$\bar{I}(\Omega) = \frac{3}{8\pi}\frac{\Gamma}{\Gamma_{\mathbf{k}_0}}\left(1-(\mathbf{n}_{eg}\mathbf{n}_\Omega)^2\right), \qquad \text{(C6)}$$

where Γ is the single atom radiative decay rate, $\Gamma_{\mathbf{k}_0}$ is the collective decay rate, $\mathbf{n}_{eg}$ is the unit vector of the atomic transition, and $\mathbf{n}_\Omega$ is a unit vector in the direction defined by the solid angle Ω. The factor $f(\Omega)$ in $I(\Omega)$ describes the interference between the emission from different atoms, and is given by Eq. (5).

Below we deduce the master equation which leads to (C4) and we sketch its solution in the case of collective states with a single excited atom.

APPENDIX D: MASTER EQUATION

The master equation for the reduced density matrix of the internal levels, which describes the ratiative decay of a set of atoms under the coupling to the quantized radiation field given by Eq. (C1), is

$$\begin{aligned}\partial_t\rho &= \sum_{i,j}\frac{\Gamma_{ij}}{2}\left(2\,\sigma_i\rho\sigma^\dagger_j - \sigma^\dagger_i\sigma_j\rho - \rho\sigma^\dagger_i\sigma_j\right)\\ &+ \frac{i}{2}\sum_{ij}G_{ij}[\sigma^\dagger_i\sigma_j,\rho], \qquad \text{(D1)}\end{aligned}$$

where the coupling constants depend on

$$\begin{aligned}J_{ij} &= \int_0^\infty d\tau g_{ij}(\tau)e^{-i\omega_L\tau} =\\ &= g^2\int_0^\infty\sum_{\mathbf{k},\lambda}\frac{\hbar\omega_k}{2\epsilon_0 V}\left(\epsilon^\lambda_{\mathbf{k}}\mathbf{d}_{ag}\right)^2 e^{i(\omega_k-\omega_L)\tau+i(\mathbf{k}-\mathbf{k}_L)\mathbf{r}}, \qquad \text{(D2)}\end{aligned}$$

in the following way:

$$\begin{aligned}\Re(J_{ij}) &= \frac{1}{2}\Gamma_{ij},\\ \Im(J_{ij}) &= \frac{1}{2}G_{ij}. \qquad \text{(D3)}\end{aligned}$$

The master equation (D1) can be solved for the particular case of an initial state (C3) by noticing that the evolution of ρ is closed within the subspace spanned by the states $|\mathbf{k}_0\rangle$, $|0\rangle$. This fact can be easily proved by direct substitution of $\rho(0) = |\mathbf{k}_0\rangle\langle\mathbf{k}_0|$ in Eq. (D1), which yields the following evolution for the atomic density matrix:

$$\rho(t) = e^{-\Gamma_{\mathbf{k}_0}t}|\mathbf{k}_0\rangle\langle\mathbf{k}_0| + \left(1-e^{-\Gamma_{\mathbf{k}_0}t}\right)|0\rangle\langle 0|, \qquad \text{(D4)}$$

where the collective decay rate $\Gamma_{\mathbf{k}_0}$ is just the Fourier transform of the coupling constants in the master equation,

$$\Gamma_{\mathbf{k}_0} = \sum_j \Gamma_{i,j}e^{i\mathbf{k}_0(\mathbf{r}^0_i-\mathbf{r}^0_j)}. \qquad \text{(D5)}$$

A similar result holds for nondiagonal elements of $\rho(t)$, which together with the quantum regression theorem yields the evolution of the atomic correlation function (C4).

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